Teacher's Content Document to Accompany the Lesson Plan:

Hit the Target! Exploration of Projectile Motion

1) Physics behind Projectile Motion

Objects **visible** to humans largely behave according to **Newton's laws of motion.** The fact that we are constraining this discussion to visible objects means that we can ignore quantum mechanics – and thus can utilize the **only** basic mathematics of geometry, trigonometry and algebra.

Objects launched at the Earth's surface are called **projectiles**.

Projectiles exhibit a discernible **path** when launched. These paths are in the shape of **parabolas**. Even if the projectiles are the same in every way, namely, weight, size and shape, the parabolic paths are **different** according to two principle variables – **initial velocity** and the **angle of firing**.

According to Newton's **First** Law, an object in straight line motion tends to stay in that same straight line motion, **unless** there is a force acting upon that object.

The main take-away from that statement is that – as the paths of projectiles are not straight lines, we know there is a **force** acting on the projectile – otherwise, you can draw the projectile's path with just a straight edge ruler. The force here is the force of Earth's **gravity**. Forces are represented by **vectors** – vectors have **magnitude** and **direction**. The direction of gravity as a force points **straight down** – towards the center of the Earth. For a projectile, there is no other force at work (that is, if we ignore the force of air friction).

Now, it has been appreciated that forces can be **decomposed** into different (usually orthogonal) **directions**. In the case of projectiles, the two main directions are: Horizontal and Vertical.

In the **Horizontal direction**, there is NO force at play. The result is that during the time the projectile is in the air, its horizontal velocity remains **constant**.

In the **Vertical direction**, gravity is constantly pulling the projectile **downwards**.

It is through this **decomposition** of analyzing projectile motion that we can predict and explain projectile motion. For an example, let's say the **initial velocity** of the projectile is 10 meters per second at a given angle, say 30 degrees above the horizontal. Then, the initial velocity (which is

also a vector with a magnitude and direction) can be decomposed, just like the acceleration vector of gravity. We can call these decomposed components (1) horizontal velocity and (2) vertical velocity.

As mentioned above, as there is **NO** force in the Horizontal direction, the horizontal velocity does **NOT** change the whole time the projectile is in the air.

The Vertical velocity is **ALWAYS** changing the whole time the projectile is in the air. For example, throw any object straight up. As you give it an initial vertical velocity towards the sky, you will see that object **SLOW** down going **UP**, and **SPEEDING** up coming **DOWN**.

The initial Vertical velocity determines the amount of time the object is in the air (that is, before it hits the ground again). This time is called **Time of Flight (TOF)**.

The initial Horizontal velocity (plus TOF) determines how far the projectile will go from the source of launch (like a cannon or some other firing mechanism). This distance is called the **RANGE**.

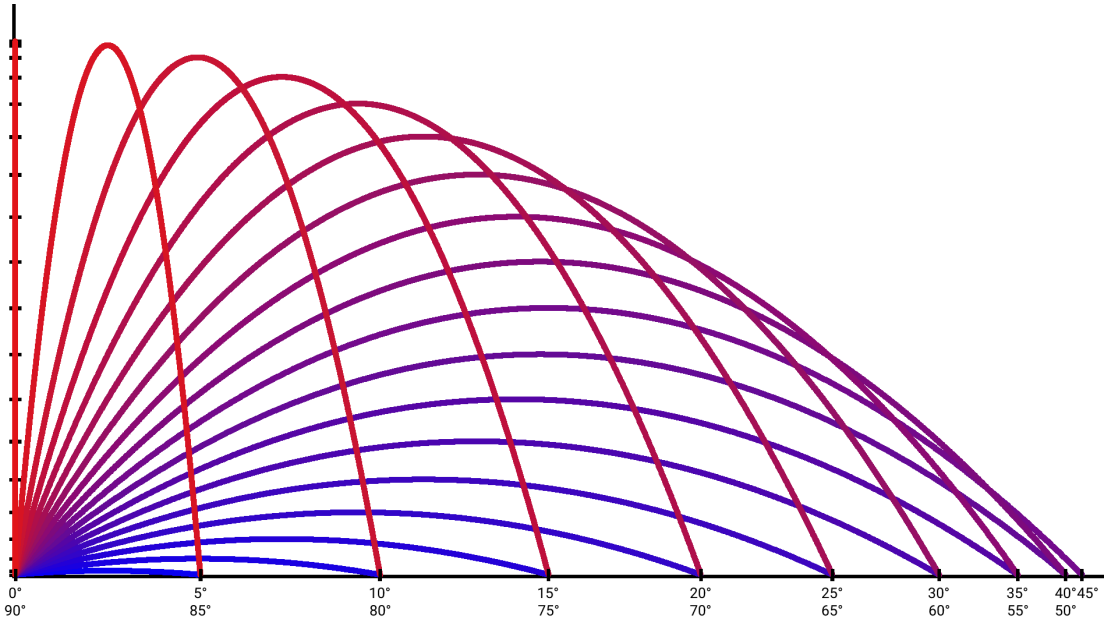
TOF is dependent on the (1) **magnitude** of the initial velocity and (2) the **angle** of launch.

Given all of the above discussion, here are a couple of rules of thumb:

1) For a given initial velocity magnitude, TOF is **maximum** where the angle is **90 degrees** pointing straight up. This makes sense – since ALL of the velocity is being used to move away from the Earth. In this case, the **RANGE** is **Zero** – since NONE of the velocity is being used to move away from the source of launch in any horizontal direction. This is the **ONLY** projectile path that is NOT a parabola. It is just a straight line – going up and coming back down the same way.
2) For a given initial velocity, the RANGE is **maximum** when the angle of launch is **45 degrees**. Please see the Youtube video, linked in the lesson plan.
3) As the angle of launch goes from 0 to 45 degrees, the **RANGE** gets longer and the maximum **HEIGHT** gets longer.
4) As the angle of launch goes from 45 to 90 degrees, the **RANGE** gets shorter (from the maximum when the angle is 45 degrees), but the **HEIGHT** continues to get longer.

The following graph summarizes these rules of thumb:

FIG. 1 – PROJECTILE PATHS USING **SAME** INITIAL VELOCITY BUT **DIFFERENT** ANGLES OF LAUNCH



[NOTE TO **DELMAR** – PLEASE CREATE A WORKSHEET FOR THE STUDENTS TO COPY OVER THEIR PATHS THAT THEY NOTE WHEN THEY ADD THE **GRID** TO THE SIMULATOR…]

The **TOP** figure (FIG. 2a) illustrates the projectile path when the **angle** of launch is **CONSTANT**, but the **initial velocity** is **DIFFERENT**. The BOTTOM figure (FIG. 2b) is just like FIG. 1 above.

FIG. 2(a) – PROJECTILE PATHS WITH **SAME** ANGLE, BUT **DIFFERENT** INITIAL VELOCITIES

